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Marginal deformations and closed string couplings in open string field theory

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Abstract: We investigate analytic classical solutions in open string field theory which are constructed in terms of marginal operators. In the classical background, we evaluate a coupling between an on-shell closed string state and the open string field. The resulting coupling exhibits periodic behavior as expected from the marginal boundary deformation of background Wilson lines or a marginal tachyon lump. We confirm that the solutions in open string field theory correspond to a class of marginal deformations in conformal field theory.

Keywords: Bosonic Strings, D-branes, String Field Theory.

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Contents

1. Introduction

String field theory possesses enormously large gauge structure which can never be found in low energy effective field theories. String field, its integration and the BRS charge have similar properties to a connection, the integration of differential forms and the exterior derivative in ordinary differential geometry. The action of open string field theory is analogue to the integration of the Chern-Simons three form, and then the theory is invariant under analogous gauge symmetry [1, 2]. Moreover, the gauge symmetry becomes much larger than that of effective theories due to infinite component fields of string field and their product with complicated non-local structure.

The equation of motion in open string field theory means that its solutions can be expressed as an analogue to a flat connection in differential geometry [1]. By analogy with field theory, we expect that classical solutions can be classified by topologically inequivalent gauge group elements in string field theory. For example, the tachyon vacuum solution should be obtained as a kind of large gauge transformations of the perturbative vacuum as proposed in ref. [3]. Hence, gauge structure of string field theory may be an important and fundamental concept for the classification of non-perturbative vacua in string field theory.

It was proposed in refs. [3] and [4] that an analytic classical solution is represented as a locally pure gauge configuration in open string field theory, and it can be related to a marginal boundary deformation in conformal field theory. This correspondence between gauge transformations and marginal deformations is a natural generalization of that of effective theories. On the other hand, using the level truncation analysis [9], we can construct a one parameter family of classical solutions associated with the marginal operator. Naturally, we expect that these are physically equivalent solutions and that the solution in the level truncated theory may be expressed also as locally pure gauge form. However, it is difficult to compare these solutions directly and to assure their equivalence because of their gauge difference. In addition, it is impossible to rewrite the level truncated solution as a pure gauge form because the level truncation breaks gauge symmetry of string field theory.

Though the level truncated theory is not helpful to find the gauge structure, the theory is very useful to evaluate vacuum energy for some classical solutions [5, 6, 7, 8]. Actually, we find that the vacuum energy of the marginal solution approaches to zero for all allowed value of the massless field as the truncation level is increased [9]. This result is consistent with the fact that the marginal solution corresponds to a boundary marginal deformation in conformal field theory. In contrast to the truncated case, it turns out that the vacuum energy of the analytic solution is a kind of indefinite quantities [10]. Thus, each one of the solutions has both merits and demerits.

In order to understand both gauge group and vacuum structures in string field theory, we should clarify further the relation of the analytic and truncated solutions, as a first step, for marginal deformations. For the level truncated solution, it has recently become possible to construct the energy momentum tensor associated with the marginal solution and to compare this with the result in conformal field theory, namely a disk amplitude with a graviton vertex insertion in the marginal deformed background [11]. If these solutions are gauge equivalent, we may provide similar results for the analytic classical solution.

In this paper, we will calculate open-closed string couplings around the vacuum corresponding to the analytic marginal solutions. We use gauge invariant operators proposed in ref. [12] to represent the open-closed string coupling which is related to a disk amplitude with an additional open string vertex insertion. As a result, we reproduce their expected periodic behavior for marginal parameters, for example creation and annihilation of an array of D-branes [13, 14, 15]. These results confirm that the analytic marginal solutions are associated with marginal boundary deformations of the conformal field theory.

In section 2 we will consider a class of analytic solutions corresponding to Wilson lines background and marginal tachyon lumps as in refs. [3, 4]. In section 3 we will derive oscillator expression of the the on-shell closed string coupling to the open string field. Using this expression, we will evaluate open-closed string couplings in several classical backgrounds.

2. Classical solutions and marginal deformations

The action in open string field theory is given by [1]

$$S[\Psi] = -\frac{1}{g^2}\int\left(\frac{1}{2}\Psi * Q_{\rm B}\Psi + \frac{1}{3}\Psi * \Psi * \Psi\right). \qquad (2.1)$$

Under the gauge transformation $\Psi = U * Q_{\rm B}\, U^{-1} + U * \Psi' * U^{-1}$, the action is transformed as $S[\Psi] = S[U * Q_{\rm B}\, U^{-1}] + S[\Psi']$. The functional U is an element of the gauge group in which the multiplication law is given by the star product. We expect that the action is invariant under gauge transformations implemented by the gauge functional which is homotopically equivalent to the identity string field I. Because such gauge functionals can be generated by

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integrating infinitesimal gauge transformations from the identity, and the action is invariant under the infinitesimal gauge transformation $\delta\Psi = Q_{\rm B}\Lambda + \Psi * \Lambda - \Lambda * \Psi$ [16, 17, 18]. From variational principle, the equation of motion becomes $Q_{\rm B}\Psi + \Psi * \Psi = 0$.

We consider the case of no Chan-Paton degrees of freedom. We single out a direction and write its string coordinate as $X(z, \bar{z}) = (X(z) + X(\bar{z}))/2$. Then we can construct a classical solution using the operator $\partial X(z)$ [3, 4]:

$$\Psi_0(\lambda) = -\lambda V_L(F)I + \frac{1}{4}\lambda^2 C_L(F^2)I\,, \tag{2.2}$$

where λ is a real parameter and the operators $V_L(F)$ and $C_L(F^2)$ are defined as

$$V_L(F) = \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z)\, \frac{i}{2\sqrt{\alpha'}} c(z)\partial X(z)\,, \qquad C_L(F^2) = \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z)^2\, c(z)\,. \tag{2.3}$$

Here $C_{\rm left}$ denotes the integration path along the left half of a string and $F(z)$ is a function satisfying $F(-1/z) = z^2 F(z)$ and $F(\pm i) = 0$.[1] The function $F(z)$ corresponds to $F_+^{(1)}(z)$ in ref. [3] and we will see later that $F_+^{(1)}(z)$ provides a solution with the same physical property as $F_-^{(1)}(z)$.[2]

We introduce an operator using $X(z)$ and $F(z)$:

$$X_L(F) = \int_{C_{\rm left}} \frac{dz}{2\pi i} F(z) X(z). \tag{2.7}$$

Using properties of $X_L(F)$ in ref. [3], we can rewrite locally the solution as a pure gauge form:

$$\Psi_0(\lambda) = U(\lambda) * Q_{\rm B}\, U(\lambda)^{-1}, \tag{2.8}$$

where the group element $U(\lambda)$ is

$$U(\lambda) = \exp\left(\frac{i\,\lambda}{2\sqrt{\alpha'}}\, X_L(F)\, I\right), \tag{2.9}$$

and we have defined as $\exp A = I + A + A * A/2! + \cdots$. This is a local expression because the operator $X_L(F)$ includes the zero mode $\hat{x}$ of the string coordinate.

[1] We denote the complex coordinate w in refs. [3, 4] by z.

[2] We can show that $\Psi_0(\lambda)$ obeys the equation of motion by using the commutation relation

$$\{V_L(F),\, V_L(F)\} = -\frac{1}{2}\{Q_{\rm B},\, C_L(F^2)\}, \tag{2.4}$$

and the following properties

$$\begin{aligned} (V_R(F)A) * B &= -(-1)^{|A|} A * V_L(F)B\,, \\ (C_R(F)A) * B &= -(-1)^{|A|} A * C_L(F)B\,, \qquad (2.5) \\ V_R(F)I &= -V_L(F)I\,, \qquad C_R(F)I = -C_L(F)I\,. \qquad (2.6) \end{aligned}$$

If we expand the string field as $\Psi = \Psi_0(\lambda) + \tilde{\Psi}$, the action becomes

$$S[\Psi] = S[\Psi_0(\lambda)] - \frac{1}{g^2}\int\left(\frac{1}{2}\tilde{\Psi} * Q'_{\mathrm{B}}(\lambda)\tilde{\Psi} + \frac{1}{3}\tilde{\Psi} * \tilde{\Psi} * \tilde{\Psi}\right). \tag{2.10}$$

The modified BRS charge $Q'_{\mathrm{B}}(\lambda)$ is given by

$$Q'_{\mathrm{B}}(\lambda) = Q_{\mathrm{B}} - \lambda\left(V_L(F) + V_R(F)\right) + \frac{\lambda^2}{4}\left(C_L(F^2) + C_R(F^2)\right), \tag{2.11}$$

where the operators $V_R(F)$ and $C_R(F^2)$ are defined by replacing C_{left} in (2.3) by C_{right} which is the path along the right half of a string. As discussed in ref. [3], the action for the quantum fluctuation is transformed to the original form by the string field redefinition:

$$\begin{aligned}\tilde{\Psi} &= e^{B(\lambda)}\Psi', \\ B(\lambda) &= \frac{i\lambda}{2\sqrt{\alpha'}}X_L(F) + \frac{i\lambda}{2\sqrt{\alpha'}}X_R(F),\end{aligned} \tag{2.12}$$

where the operator $X_R(F)$ is defined by replacing C_{left} in (2.7) by C_{right}. For this redefinition, one of important equations is $Q_{\mathrm{B}} = e^{-B(\lambda)}Q'_{\mathrm{B}}(\lambda)e^{B(\lambda)}$. Thus, the action $S[\Psi]$ is transformed to the original form $S[\Psi']$ with the constant term $S[\Psi_0(\lambda)]$. This result suggests that the string field expansion and the redefinition can be regarded as a gauge transformation in string field theory. Indeed, the expansion and redefinition of the string field can be written locally in terms of the gauge group element (2.9):

$$\begin{aligned}\Psi &= \Psi_0(\lambda) + e^{B(\lambda)}\Psi' \\ &= U(\lambda) * Q_{\mathrm{B}}\, U(\lambda)^{-1} + U(\lambda) * \Psi' * U(\lambda)^{-1}.\end{aligned} \tag{2.13}$$

To exhibit the physical meaning of the classical solution, we choose the function $F(z) = (z + 1/z)/z$ as the simplest case. Though there is other possibility for $F(z)$, we can transform the solution for other functions to this simplest case by string field redefinitions or gauge transformations. In this case, the solution (2.2) has a well-defined Fock space expression [3, 4]:

$$|\Psi_0(\lambda)\rangle = \frac{\lambda^2}{2\pi}c_1\,|0\rangle - \frac{\lambda}{\sqrt{2\pi}}c_1\alpha_{-1}\,|0\rangle + \frac{\lambda^2}{6\pi}c_{-1}\,|0\rangle + \frac{\lambda^2}{2\pi}c_1 L_{-2}\,|0\rangle + \cdots, \tag{2.14}$$

and the operator $B(\lambda)$ can be written

$$B(\lambda) = -\frac{\lambda}{\sqrt{2}}(\alpha_1 - \alpha_{-1}). \tag{2.15}$$

Here, α_n and L_n are oscillators of $X(z)$ and total Virasoro generators, respectively, and $|0\rangle$ is the $\mathrm{SL}(2,\mathbb{R})$ invariant vacuum and the abbreviation denotes higher level states. Using oscillator representation, we write the string field as

$$|\Psi\rangle = \phi(x)\,c_1\,|0\rangle + A(x)\,c_1\alpha_{-1}\,|0\rangle + A_i(x)\,c_1\alpha^i_{-1}\,|0\rangle + \cdots, \tag{2.16}$$

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where α_n^i $(i = 0, \ldots, 24)$ is oscillators of other directions except $X(z)$ and the abbreviation denotes higher level component fields. Substituting (2.15) into (2.12), we can represent the string field redefinition using component fields:

$$\tilde{\phi}(x) = e^{-\frac{\lambda^2}{4}} \phi'(x) - \frac{\lambda}{\sqrt{2}} e^{-\frac{\lambda^2}{4}} A'(x) + \cdots, \tag{2.17}$$

$$\tilde{A}(x) = \frac{\lambda}{\sqrt{2}} e^{-\frac{\lambda^2}{4}} \phi'(x) + \left(1 - \frac{\lambda^2}{2}\right) e^{-\frac{\lambda^2}{4}} A'(x) + \cdots, \tag{2.18}$$

$$\tilde{A}_i(x) = e^{-\frac{\lambda^2}{4}} A_i'(x) + \cdots. \tag{2.19}$$

It should be noticed that, in these equations, all coefficients of component fields have regular expression for any λ,[3] and then the string field redefinition (2.12) is well-defined for all λ.

We expect that this solution corresponds to a the marginal deformation of Wilson lines because $\Psi_0(\lambda)$ contains a vacuum expectation value of a massless vector state, $c_1 \alpha_{-1} |0\rangle$, in the expression (2.14). In fact, if we introduce Chan-Paton indices as Ψ_{ij}, the string field redefinition from $\tilde{\Psi}$ to Ψ' is written as

$$\tilde{\Psi}_{ij} = e^{B(\lambda_i, \lambda_j)} \Psi'_{ij}, \tag{2.20}$$

where the operator $B(\lambda_i, \lambda_j)$ is given by

$$\begin{aligned} B(\lambda_i, \lambda_j) &= -\frac{i\lambda_i}{2\sqrt{\alpha'}} X_L(F) - \frac{i\lambda_j}{2\sqrt{\alpha'}} X_R(F), \\ &= i\, \frac{\lambda_i - \lambda_j}{\pi\sqrt{\alpha'}} \hat{x} - \frac{\lambda_i + \lambda_j}{2\sqrt{2}} (\alpha_1 - \alpha_{-1}) + \cdots. \end{aligned} \tag{2.21}$$

Hence, this string field redefinition changes the momentum of $\tilde{\Psi}_{ij}$ as $p \to p + (\lambda_i - \lambda_j)/\pi\sqrt{\alpha'}$ and this is the same effect as background Wilson lines [3, 4].

Since the analytic solution $\Psi_0(\lambda)$ corresponds to the marginal deformation, the vacuum energy $S[\Psi_0(\lambda)]$ should be zero as expected from that of the level truncated solution in ref. [9]. Formally, we can evaluate the vacuum energy in the following [19]. We differentiate $S[\Psi_0(\lambda)]$ with respect to λ:

$$\frac{d}{d\lambda} S[\Psi_0(\lambda)] = -\frac{1}{g^2} \int \frac{d\Psi_0(\lambda)}{d\lambda} * (Q_{\mathrm{B}} \Psi_0(\lambda) + \Psi_0(\lambda) * \Psi_0(\lambda)) = 0, \tag{2.22}$$

where we have used the fact that $\Psi_0(\lambda)$ obeys the equation of motion. Then, we find that $S[\Psi_0(\lambda)]$ is a constant and independent of λ. Since $\Psi_0(0) = 0$ and then $S[\Psi_0(0)] = 0$, we verify that the vacuum energy becomes zero, that is $S[\Psi_0(\lambda)] = 0$. However, if we evaluate $S[\Psi_0(\lambda)]$ directly in terms of its oscillator representation, we find that the vacuum energy is an indefinite quantity given by zero from the ghost sector times infinity from the matter sector as discussed in ref. [10]. Then, there should be exist a kind of regularization method to calculate the vacuum energy directly. Though it is an open question to find the regularization, we will formally handle similar indefinite quantities in the next section.

[3]This result follows from the fact that a normal ordered form of the operator $\exp B(\lambda)$ is well-defined regular expression for any λ.

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We can construct other analytic classical solutions corresponding to different marginal deformations. As an example, we consider the case that the direction X is compactified on a circle of critical radius. At the critical radius, there are the following three conserved currents

$$J^1(z) = \frac{i}{2\sqrt{\alpha'}}\partial X(z)\,, \tag{2.23}$$

$$J^2(z) = \frac{1}{2}\left(e^{\frac{i}{\sqrt{\alpha'}}X(z)} + e^{-\frac{i}{\sqrt{\alpha'}}X(z)}\right), \tag{2.24}$$

$$J^3(z) = \frac{1}{2i}\left(e^{\frac{i}{\sqrt{\alpha'}}X(z)} - e^{-\frac{i}{\sqrt{\alpha'}}X(z)}\right), \tag{2.25}$$

and these currents generate a SU(2) symmetry in string theory. If we expand the currents as[4]

$$J^2(z) = \frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty}\beta_n z^{-n-1}\,, \qquad J^3(z) = \frac{1}{\sqrt{2}}\sum_{n=-\infty}^{\infty}\gamma_n z^{-n-1}\,, \tag{2.26}$$

these oscillators satisfy the commutation relations

$$\begin{aligned}
&[\alpha_m,\,\alpha_n] = m\delta_{m+n,0}\,, \qquad &&[\beta_m,\,\beta_n] = m\delta_{m+n,0}\,, \qquad &&[\gamma_m,\,\gamma_n] = m\delta_{m+n,0}\,,\\
&[\alpha_m,\,\beta_n] = i\,\gamma_{m+n}\,, &&[\beta_m,\,\gamma_n] = i\,\alpha_{m+n}\,, &&[\gamma_m,\,\alpha_n] = i\,\beta_{m+n}\,.
\end{aligned} \tag{2.27}$$

It can be easily seen that α_0, β_0 and γ_0 commute with the BRS charge and these zero modes are conserved on the three string vertex in the action. Then, the action (2.1) is invariant under the SU(2) rotation

$$\Psi = \exp\left(i\theta_1\alpha_0 + i\theta_2\beta_0 + i\theta_3\gamma_0\right)\,\Psi'\,, \tag{2.28}$$

where θ_k are real parameters. Acting a SU(2) rotation on (2.2), we can find as an analytic solution,

$$\Psi_0^k(\lambda) = -\lambda V_L^k(F)I + \frac{\lambda^2}{4}C_L(F^2)I\,, \tag{2.29}$$

where $V_L^k(F)$ are defined by

$$V_L^k(F) = \int_{C_{\rm left}}\frac{dz}{2\pi i}F(z)\,c(z)\,J^k(z) \qquad (k=1,2,3)\,. \tag{2.30}$$

The $k=1$ case corresponds to the previous marginal configuration. If we choose $F(z) = (z+1/z)/z$ and we expand the string field as $\Psi = \Psi_0^2(\lambda) + \tilde{\Psi}$, the action of the fluctuation goes back to the original action by the string field redefinition

$$\tilde{\Psi} = e^{B'(\lambda)}\Psi'\,, \qquad B'(\lambda) = -\frac{\lambda}{\sqrt{2}}(\beta_1 - \beta_{-1})\,. \tag{2.31}$$

Instead of use of the SU(2) rotation, we can show more explicitly that the solution (2.29) obeys the equation of motion as discussed in refs. [3, 4].

[4]Using oscillators of $X(z)$, the current $J^1(z)$ is expanded as $J^1(z) = \sum_n \alpha_n z^{-n-1}/\sqrt{2}$.

3. Closed string couplings and marginal deformations

In this section we will consider open-closed string couplings at the classical vacuum corresponding to the analytic marginal solutions. As in refs. [12, 20, 21, 22, 23], we can incorporate closed strings by introducing the term

$$\langle V|\,|\Psi\rangle\,, \qquad \langle V| = \langle I|\,V\left(\frac{\pi}{2}\right), \tag{3.1}$$

where $V(\sigma)$ corresponds to an on-shell closed string vertex operator $V(\sigma) = c_+(\sigma)c_-(\sigma)\times O(\sigma)$. We consider the case that the direction X is compactified on a circle of radius R and the Neumann boundary condition is imposed on X. If the vertex operator has no derivative operator of this direction, namely $\partial X, \partial^2 X, \ldots$, the closed string state with the momentum m/R and the winding number w is given by

$$\langle V(m,w;R)| = \langle V_{c=25}| \otimes \langle m,w;R| \otimes \langle V_{gh}|\,, \tag{3.2}$$

where $\langle V_{c=25}|$ and $\langle V_{gh}|$ correspond to vertex operators of the rest of the $c = 25$ CFT and the ghost CFT. The state $\langle m,w;R|$ is given by the definition which refers to the $c = 1$ CFT of the compactified direction:

$$\langle m,w;R|\,|\phi\rangle = \left\langle e^{ik_L X(i)+ik_R X(-i)}\, h[\phi(0)]\right\rangle, \tag{3.3}$$

where the state $|\phi\rangle$ is given in the form $|\phi\rangle = \phi(z=0)\,|0\rangle$ for an operator $\phi(z)$, and $h[\cdots]$ denotes the conformal mapping corresponding to the function $u = h(z) = 2z/(1-z^2)$, which maps the unit disk $|z| \leq 1$ into the whole complex u-plane. The correlation function of the right hand-side is defined in the u-plane and the momenta k_L and k_R are given by $k_L = (m/R + wR/\alpha')/2$ and $k_R = (m/R - wR/\alpha')/2$.

From the definition (3.3), we can write the state $\langle m,w;R|$ by using oscillator expression:

$$\langle m,w;R| = N\left\langle -\frac{m}{R}\right| \exp\sum_{n=1}^{\infty}(-1)^n\left\{-\frac{1}{2n}\alpha_n\alpha_n - \frac{\sqrt{2\alpha'}}{n}\frac{m}{R}\alpha_{2n} - \frac{2i\sqrt{2\alpha'}}{2n-1}\frac{wR}{\alpha'}\alpha_{2n-1}\right\}, \tag{3.4}$$

where N denotes a normalization factor depending on m, w and R, and the state $\langle -m/R|$ denotes the eigenstate of $\hat{p}$, that is $\langle -m/R|\,\hat{p} = -m/R\,\langle -m/R|$. The derivation is put in appendix A. From the oscillator expression, we observe that the momentum is conserved in this open-closed string coupling but the winding number is not because the closed string state contains only the zero mode eigenstate $\langle -m/R|$. We can obtain a closed string coupling to a Dirichlet open string by the T-dual transformation, $m \leftrightarrow w$ and $R \leftrightarrow \alpha'/R'$. This closed string state is represented by

$$\left\langle w,m;\frac{\alpha'}{R'}\right| = \left\langle -\frac{wR'}{\alpha'}\right| \exp\sum_{n=1}^{\infty}(-1)^n\left\{-\frac{1}{2n}\alpha_n\alpha_n - \frac{\sqrt{2\alpha'}}{n}\frac{wR'}{\alpha'}\alpha_{2n} - \frac{2i\sqrt{2\alpha'}}{2n-1}\frac{m}{R'}\alpha_{2n-1}\right\}. \tag{3.5}$$

In this T-dual picture, the winding number is conserved but the momentum is not. This is a consistent result with the coupling of a closed string in a bulk and an open string on a D-brane.

Now, let us consider open-closed string couplings at the classical background corresponding to the solution (2.2):

$$\langle V(m,w;R)|\,|\Psi\rangle = \langle V(m,w;R)|\,|\Psi_0(\lambda)\rangle + \langle V(m,w;R)|\,e^{B(\lambda)}\,|\Psi'\rangle\,. \tag{3.6}$$

Note that the action of Ψ' has the original form with the ordinary BRS charge as discussed in the previous section. First, we can show that the inhomogeneous term $\langle V(m,w;R)|\,|\Psi_0(\lambda)\rangle$ vanishes in the following. Differentiating the solution $\Psi_0(\lambda)$ with respect to λ, we find

$$\frac{d}{d\lambda}\,|\Psi_0(\lambda)\rangle = -Q'_{\rm B}(\lambda)\,\frac{i}{2\sqrt{\alpha'}}X_L(F)\,|I\rangle\,, \tag{3.7}$$

where we have used commutation relations of V_L, C_L and X_L. Moreover, using the oscillator expression of the closed string state, we can find the modified BRS invariance of $\langle V(m,w;R)|$:

$$\langle V(m,w;R)|\,Q'_{\rm B}(\lambda) = 0\,. \tag{3.8}$$

The proof is given in appendix B. From (3.7) and (3.8), we obtain a similar equation to that of the vacuum energy (2.22):

$$\frac{d}{d\lambda}\,\langle V(m,w;R)|\,|\Psi_0(\lambda)\rangle = 0\,. \tag{3.9}$$

Similarly, we verify that the term $\langle V(m,w;R)|\,|\Psi_0(\lambda)\rangle$ is to be zero irrelevant to λ. Here, it should be noticed that this evaluation is rather formal as well as that of the vacuum energy. From direct calculation using oscillator expression, we find that this term also is an indefinite quantity. This question remains open.

As a result, at the classical background, the closed string state is changed by the transformation generated by $B(\lambda)$ from the original form (3.2). Since the operator $B(\lambda)$ contains oscillators of only the compactified direction, we have only to calculate the effect of $B(\lambda)$ on $\langle m,n;R|$ in order to evaluate the right hand-side of (3.6). For the function $F(z)=(z+1/z)/z$, using the oscillator expression (2.15) and (3.4), we can easily find that

$$\langle m,w;R|\,B(\lambda) = i\frac{wR}{\alpha'}\,2\sqrt{\alpha'}\lambda\,\langle m,w;R|\,. \tag{3.10}$$

Then, we obtain a final expression of the open-closed string coupling:

$$\langle V(m,w;R)|\,|\Psi\rangle = \exp\left(i\frac{wR}{\alpha'}\,2\sqrt{\alpha'}\lambda\right)\langle V(m,w;R)|\,|\Psi'\rangle\,. \tag{3.11}$$

Thus, the classical solution for the marginal deformation causes the phase shift which is proportional to the winding number in the open-closed strings coupling. This phase shift (3.11) completely agrees with the effect of the marginal deformation of the Wilson line flux [14]. In the T-dual picture, the coupling is rewritten as

$$\left\langle V\left(w,m;\frac{\alpha'}{R'}\right)\right|\,|\Psi\rangle = \exp\left(i\frac{m}{R'}\,2\sqrt{\alpha'}\lambda\right)\left\langle V\left(w,m;\frac{\alpha'}{R'}\right)\right|\,|\Psi'\rangle\,. \tag{3.12}$$

This implies that the marginal deformation moves the position of D-brane as usual [14].

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Now consider how the closed string coupling at the critical radius is changed by string field expansion around Ψ_0^2 and the string field redefinition (2.31). In this case too, we find that the inhomogeneous term becomes zero for any λ. Then, we have only to consider the effect of the string field redefinition on the closed string state.

For simplicity, we consider the case that the closed string state is given by $\langle V| = \langle V(0,1)| - \langle V(0,-1)|$, where we have used $\alpha' = 1$ unit and omitted the radius $R = \sqrt{\alpha'}$ in the expression. From the definition (3.3) , the compactified sector of $\langle V(0,1)|$ is defined by a midpoint insertion of the vertex operator

$$\mathcal{V}(u,\bar{u}) = e^{\frac{i}{2}X(u)-\frac{i}{2}X(\bar{u})}\,. \tag{3.13}$$

The effect of γ_0 on $\langle V(0,1)|$ can be evaluated by deforming the integration path of γ_0 to contours around $\pm i$ in the u plane and calculating OPE:

$$-\left(\oint_{C_i}\frac{du}{2\pi i}\,J^3(u)+\oint_{C_{-i}}\frac{du}{2\pi i}\,J^3(u)\right)\mathcal{V}(i,-i) = -\frac{i}{2}e^{-\frac{i}{2}X(i)-\frac{i}{2}X(-i)}+\frac{i}{2}e^{\frac{i}{2}X(i)+\frac{i}{2}X(-i)}\,, \tag{3.14}$$

and then we find

$$\langle V(0,1)|\,\gamma_0 = \frac{i}{2}\,\langle V(1,0)| - \frac{i}{2}\,\langle V(-1,0)|\,. \tag{3.15}$$

From this and similar equations, we obtain

$$\begin{aligned}\langle V(0,\pm 1)|\,e^{i\theta\gamma_0} &= \cos^2\frac{\theta}{2}\,\langle V(0,\pm 1)| - \sin^2\frac{\theta}{2}\,\langle V(0,\mp 1)| - \\ &\quad -\frac{1}{2}\sin\theta\,\{\langle V(1,0)| - \langle V(-1,0)|\}\,.\end{aligned} \tag{3.16}$$

As a result, we find that the state $\langle V| = \langle V(0,1)| - \langle V(0,-1)|$ is invariant under the rotation of γ_0. Furthermore, it can be easily seen that $\langle V|$ is a singlet state of the SU(2) symmetry generated by α_0, β_0 and γ_0. Hence, this closed string state $\langle V|$ is regarded as a superposition of states with the winding number ± 1 or with the momentum ± 1 in the T-dual picture, and this interpretation is unchanged under the SU(2) transformation.

We can rewrite $B'(\lambda)$ as a SU(2) rotation of $B(\lambda)$ by using the commutation relations (2.27):

$$B'(\lambda) = e^{-i\frac{\pi}{2}\gamma_0}\,B(\lambda)\,e^{i\frac{\pi}{2}\gamma_0}\,. \tag{3.17}$$

Then, the closed string state is transformed by the string field redefinition (2.31) to

$$\begin{aligned}\langle V|\,e^{B'(\lambda)} &= \langle V|\,e^{B(\lambda)}\,e^{i\frac{\pi}{2}\gamma_0} \\ &= e^{i\,2\lambda}\,\langle V(0,1)|\,e^{i\frac{\pi}{2}\gamma_0} - e^{-i\,2\lambda}\,\langle V(0,-1)|\,e^{i\frac{\pi}{2}\gamma_0}\,,\end{aligned} \tag{3.18}$$

where we have used the SU(2) invariance of $\langle V|$ and the transformation law (3.10). Applying (3.16), we find final expression of the transformation law of the closed string state:

$$\langle V|\,e^{B'(\lambda)} = \cos 2\lambda\,\langle V| - i\sin 2\lambda\,\langle V'|\,,\qquad \langle V'| = \langle V(1,0)| - \langle V(-1,0)|\,, \tag{3.19}$$

where $\langle V'|$ is a superposition of states with the momentum ± 1 or with the winding number ± 1 in the T-dual picture.

Now that the closed string state at the classical background is given, let us consider the interpretation of the resulting open-closed string coupling. Supposed that the closed string state has momentum but no winding number in the coupling, and then the arguments ± 1 of both of $\langle V|$ and $\langle V'|$ are to be momentum in the compactified direction. Then, the coupling $\langle V||\Psi\rangle$ can be regarded as that of the closed string state to the open string field on an array of D-branes with Dirichlet boundary condition on X, and $\langle V'||\Psi\rangle$ corresponds to a closed string coupling to the open string field with Neumann boundary condition on X. As a result, the transformation law (3.19) implies that string field condensation on an array of D-branes causes annihilation of the D-brane array at $\lambda = \pi/4$ and creation of the same D-brane array at $\lambda = \pi/2$. This effect of string field condensation is the same as that of the marginal deformation associated with the current J^2 [15].

Finally, let us reexamine the relation between the marginal deformation parameter in conformal field theory and the corresponding parameter λ in string field theory. As in the previous section, the string field condensation associated with J^1 produces the momentum shift as $p \to p + (\lambda_i - \lambda_j)/\pi\sqrt{\alpha'}$ if we introduce the Chan-Paton indices. Comparing this effect with that of conformal field theory, we conclude that string field condensation of (2.14) corresponds to adding to the world-sheet action the boundary term

$$i\,\frac{\lambda}{\pi\sqrt{\alpha'}}\int dt\,\partial_t X(t) = i\frac{\theta}{2\pi R}\int dt\,\partial_t X(t) = i\frac{\theta R'}{2\pi\alpha'}\int dt\,\partial_t X(t)\,, \tag{3.20}$$

where we introduce the parameter $\theta = 2\lambda R/\sqrt{\alpha'} = 2\sqrt{\alpha'}\lambda/R'$. From the transformation law (3.12) in the T-dual picture, it follows that this string field condensation induces the shift of D-brane position as $x \to x + 2\sqrt{\alpha'}\lambda = x + \theta R'$. This result completely agrees with the effect of the boundary term (3.20) in conformal field theory.

At the critical radius,[5] the boundary term (3.20) can be transformed to

$$\frac{2\lambda}{\pi}\int dt\ \cos X(t)\,, \tag{3.21}$$

by a SU(2) rotation. This boundary term creates or annihilate an array of D-branes at $2\lambda/\pi = 1/2$ [13, 14, 15]. This effect has also complete agreement with the result obtained from (3.19) in string field theory.

4. Discussions

We studied the analytic classical solutions in string field theory which corresponds to boundary marginal deformations in conformal field theory. In particular, we evaluated the closed string couplings to the open string field in the classical backgrounds and then we confirmed that the resulting couplings have the periodic properties expected from conformal field theory.

In the open-closed string coupling, we did not consider the vertex operator with derivatives of the compactified direction. However, we should calculate more generic closed string

[5]We have used $\alpha' = 1$ unit.

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vertex operators in order to prove the correspondence between the classical solution and the marginal deformation. At the critical radius, we should evaluate the open-closed string coupling in which the closed string state associates to a more general representation of the SU(2) group. These problems remain to be resolved.

These analytic classical solutions have the manifest relation to the marginal deformations as far as string field redefinitions and open-closed string couplings are concerned. To make the relation more precise, we must understand the vacuum energy of the classical solutions, whereas we can not evaluate it directly because of its indefiniteness. However, there is one possible method to speculate it. We can determine the vacuum energy indirectly by investigating the tachyon vacuum in the theory expanding string field around the classical solution. Since the Wilson lines flux does not affect the D-brane tension, the vacuum energy of the tachyon vacuum could be independent of the marginal parameter of the solution. We could confirm this conjecture by using the level truncation analysis as discussed in ref. [10].

In the level truncated theory, the graviton coupling directly to the D-brane has been obtained from the energy momentum tensor in string field theory [9]. This graviton coupling can not be calculated by considering gauge invariant operators as discussed in this paper. For the analytic solutions, it has been still unclear how the direct closed string coupling to the D-brane can be derived as pointed out in ref. [9].

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A. Oscillator expression of closed string states

We will derive the oscillator expression (3.4) of the state $\langle m,w;R|$. From the definition (3.3), we can represent this state as

$$\langle m,w;R| = N\left\langle \frac{-m}{R}\right| \exp\left(\frac{1}{2}\sum_{n,l=1}^{\infty}\bar{N}_{nl}\alpha_n\alpha_l + \frac{m}{R}\sum_{n=1}^{\infty}\bar{N}_n\alpha_n + \frac{wR}{\alpha'}\sum_{n=1}^{\infty}\bar{D}_n\alpha_n\right), \qquad (A.1)$$

where N denotes a normalization factor depending on m,w and R, and $\bar{N}_{nl}$, $\bar{N}_n$ and $\bar{D}_n$ are constants.

Considering the inner product between the state (A.1) and $|k/R\rangle$ and mapping it to the u plane, we can find the equation

$$N\left\langle \frac{-m}{R}\right|\left.\frac{k}{R}\right\rangle = (\partial h(0))^{\alpha'\frac{k^2}{R^2}}\left\langle e^{ik_LX(i)+ik_RX(-i)}\,e^{i\frac{k}{R}X(0)}\right\rangle. \qquad (A.2)$$

Then, we can obtain the normalization factor N by calculating the correlation function of the right hand-side.

We can calculate $\bar{N}_n$ and $\bar{D}_n$ by multiplying $\alpha_{-n}\,|k/R\rangle$ to (A.1):

$$n\left(\frac{m}{R}\bar{N}_n + \frac{wR}{\alpha'}\bar{D}_n\right) N \left\langle \frac{-m}{R}\right|\left.\frac{k}{R}\right\rangle = \\ = (\partial h(0))^{\alpha'\frac{k^2}{R^2}} \frac{i}{\sqrt{2\alpha'}} \oint_{C_0} \frac{dz}{2\pi i} z^{-n}\,\partial h(z) \left\langle e^{ik_L X(i)+ik_R X(-i)}\,\partial X(u)\, e^{i\frac{k}{R}X(0)}\right\rangle . \qquad \text{(A.3)}$$

The correlation function in the integrand can be evaluated as

$$\left\langle e^{ik_L X(i)+ik_R X(-i)}\,\partial X(u) e^{i\frac{k}{R}X(0)}\right\rangle = \\ = \left\{ i2\alpha' \frac{m}{R}\frac{1}{u(1+u^2)} + 2\alpha'\frac{wR}{\alpha'}\frac{1}{1+u^2}\right\} \left\langle e^{ik_L X(i)+ik_R X(-i)}\, e^{i\frac{k}{R}X(0)}\right\rangle , \qquad \text{(A.4)}$$

where we have used the momentum conservation $m+k=0$. Substituting (A.4) into (A.3) and using (A.2), we find the expression for $\bar{N}_n$ and $\bar{D}_n$ as

$$\bar{N}_n = -\frac{\sqrt{2\alpha'}}{n} \oint_{C_0} \frac{dz}{2\pi i} z^{-n}\,\partial h(z)\, \frac{1}{u(1+u^2)} , \qquad \text{(A.5)}$$

$$\bar{D}_n = i\frac{\sqrt{2\alpha'}}{n} \oint_{C_0} \frac{dz}{2\pi i} z^{-n}\,\partial h(z)\, \frac{1}{u(1+u^2)} . \qquad \text{(A.6)}$$

Since the mapping $u=h(z)$ is given by $h(z)=2z/(1-z^2)$, we can expand the integrands as

$$z^{-n}\,\partial h(z)\,\frac{1}{u(1+u^2)} = z^{-n-1}\frac{1-z^2}{1+z^2} = z^{-n-1}\left(1+2\sum_{n=1}^{\infty}(-1)^n z^n\right),$$

$$z^{-n}\,\partial h(z)\,\frac{1}{1+u^2} = z^{-n}\frac{2}{1+z^2} = z^{-n-1}\left(-2\sum_{n=1}^{\infty}(-1)^n z^{2n-1}\right). \qquad \text{(A.7)}$$

Then, we can obtained the final expression for $\bar{N}_n$ and $\bar{D}_n$:

$$\bar{N}_{2n} = -\frac{\sqrt{2\alpha'}}{n}, \quad \bar{D}_{2n-1} = -\frac{2i\sqrt{2\alpha'}}{2n-1}, \quad \bar{N}_{2n-1}=0, \quad \bar{D}_{2n}=0 \qquad (n=1,2,3,\ldots). \qquad \text{(A.8)}$$

For $\bar{N}_{nl}$, by multiplying the state $\alpha_{-n}\alpha_{-l}\,|k/R\rangle$ to (A.1), we find the equation

$$n\,l\left\{\bar{N}_{nl} + \left(\frac{m}{R}\bar{N}_n + \frac{wR}{\alpha'}\bar{D}_n\right)\left(\frac{m}{R}\bar{N}_l + \frac{wR}{\alpha'}\bar{D}_l\right)\right\} N \left\langle \frac{-m}{R}\right|\left.\frac{k}{R}\right\rangle = \\ = (\partial h(0))^{\alpha'\frac{k^2}{R^2}} \left(\frac{i}{\sqrt{2\alpha'}}\right)^2 \oint_{C_0}\frac{dz}{2\pi i}\oint_{C_0}\frac{d\tilde{z}}{2\pi i} z^n\,\partial h(z)\;\tilde{z}^l \partial h(\tilde{z})\times \\ \times \left\langle e^{ik_L X(i)+ik_R X(-i)}\,\partial X(u)\,\partial X(\tilde{z})\, e^{i\frac{k}{R}X(0)}\right\rangle . \qquad \text{(A.9)}$$

Calculating the correlation function, we can find the expression

$$\bar{N}_{nl} = \frac{1}{nl}\oint_{C_0}\frac{dz}{2\pi i}\oint_{C_0}\frac{d\tilde{z}}{2\pi i}\, z^n\,\partial h(z)\;\tilde{z}^l\,\partial h(\tilde{z})\frac{1}{(u-\tilde{u})^2}. \qquad \text{(A.10)}$$

Similarly, by using the expansion

$$\partial h(z)\,\partial h(\tilde z)\,\frac{1}{(u-\tilde u)^2}=\frac{1}{(z-\tilde z)^2}+\frac{1}{(1+z\tilde z)^2}=\frac{1}{(z-\tilde z)^2}-\sum_{n=1}^{\infty}n(z\tilde z)^{n-1}\,, \qquad \text{(A.11)}$$

we obtain the final expression for $\bar N_{nl}$

$$\bar N_{nl}=-\frac{(-1)^n}{n}\delta_{n,l}\,. \qquad \text{(A.12)}$$

B. Modified BRS invariance of closed string states

We will prove that the modified BRS charge (2.11) annihilates the state (3.2).

The function $F(z)$ satisfying $F(-1/z)=z^2F(z)$ can be expanded as [3]

$$F(z)=\sum_{n=1}^{\infty}f_n\left(z^n-\left(-\frac{1}{z}\right)^n\right)z^{-1}\,. \qquad \text{(B.1)}$$

Since the function $G(z)=F(z)^2$ satisfies $G(-1/z)=z^4G(z)$, we can expand $F(z)^2$ as

$$F(z)^2=\sum_{n=0}g_n\left(z^n+\left(-\frac{1}{z}\right)^n\right)z^{-2}\,. \qquad \text{(B.2)}$$

Substituting these expansions into (2.11), we can write the modified BRS charge by using oscillator expression:

$$Q'_{\rm B}(\lambda)=Q_{\rm B}-\lambda\sum_{n=1}^{\infty}f_n\left(t_n-(-1)^nt_{-n}\right)+\frac{\lambda^2}{4}\sum_{n=0}^{\infty}g_n\left(c_n+(-1)^nc_{-n}\right), \qquad \text{(B.3)}$$

where t_n denote oscillators of the current $J(z)$ and they are written as

$$t_n=\frac{1}{\sqrt2}\sum_{m=-\infty}^{\infty}c_{n+m}\alpha_{-m}\,. \qquad \text{(B.4)}$$

By definition, it can be easily seen that $\langle V(m,w;R)|$ is a BRS invariant state. Then, in order to see the modified BRS invariance (3.8), we have only to prove the following equations

$$\langle V_{gh}|\left(c_n+(-1)^nc_{-n}\right)=0\,, \qquad \text{(B.5)}$$

$$\langle m,w;R|\otimes\langle V_{gh}|\left(t_n-(-1)^nt_{-n}\right)=0\,. \qquad \text{(B.6)}$$

The first equation (B.5) can be seen by using oscillator expression of $\langle V_{gh}|$ [20, 21]:

$$\langle V_{gh}|=\langle 0|\,c_{-1}c_0\exp\sum_{n=1}^{\infty}(-1)^{n+1}c_nb_n\,. \qquad \text{(B.7)}$$

From the expression (3.4), it follows that

$$\langle m,w;R|\left(\alpha_{2n}+\alpha_{-2n}\right)=-(-1)^n\,2\sqrt{2\alpha'}\,\frac{m}{R}\,\langle m,w;R|\,, \qquad \text{(B.8)}$$

$$\langle m,w;R|\left(\alpha_{2n-1}-\alpha_{-2n+1}\right)=(-1)^n\,2i\sqrt{2\alpha'}\,\frac{wR}{\alpha'}\,\langle m,w;R|\,. \qquad \text{(B.9)}$$

Using (B.5), (B.8) and (B.9), we can calculate the left hand-side of (B.6) and find that it becomes zero. Hence, we can see that the equation (3.8) holds.

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